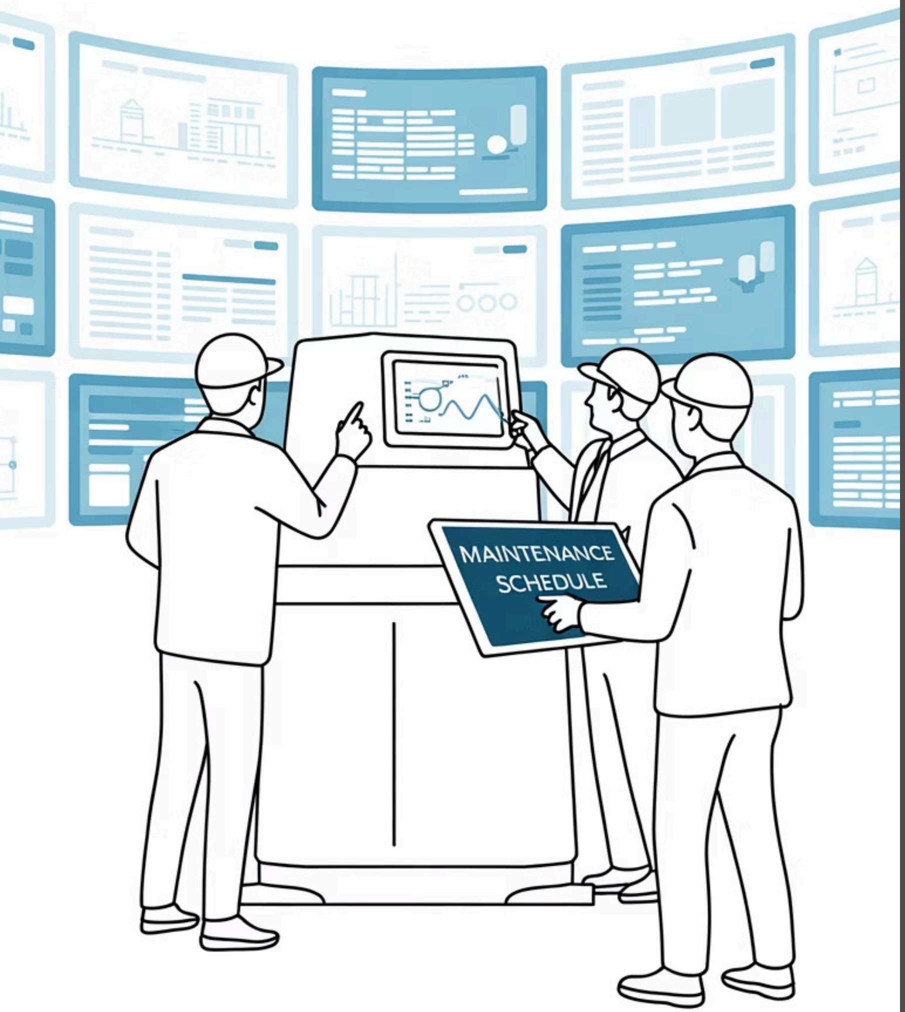
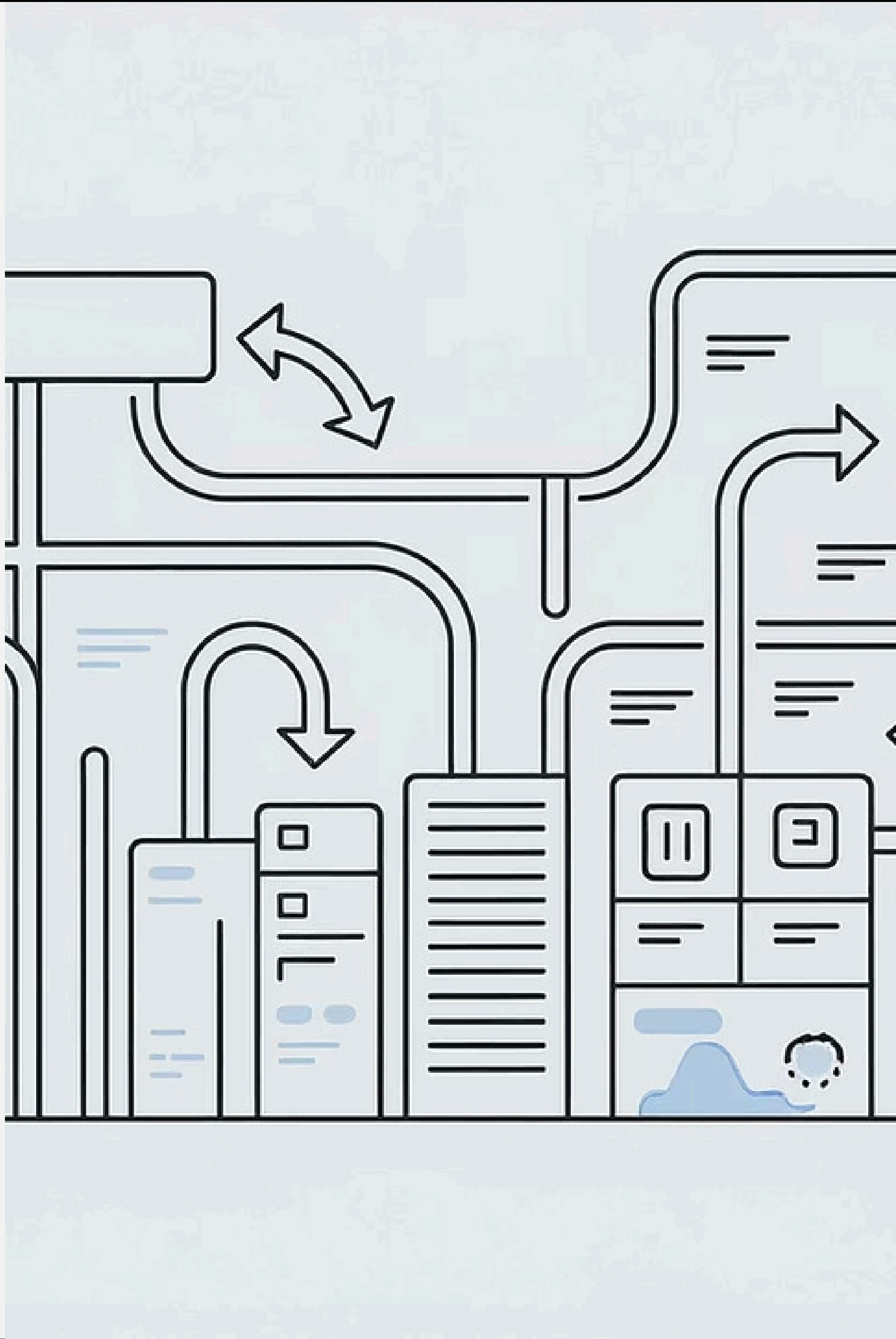


8.4.2 Exemple of algorithmuse in MODAPTO:

System Description & Problem Formulation

System Context



Series System Architecture

Our focus is on a **series system** comprised of **Nc components**, where each component represents a critical point of potential failure. In this configuration, if any single component fails, the entire system fails, making preventive maintenance particularly crucial.

Each component within the system has unique characteristics that influence maintenance planning, including:

- Component-specific reliability profiles
- Varied maintenance durations and complexity
- Different resource requirements
- Specialized technical expertise needs

Maintenance Requirements

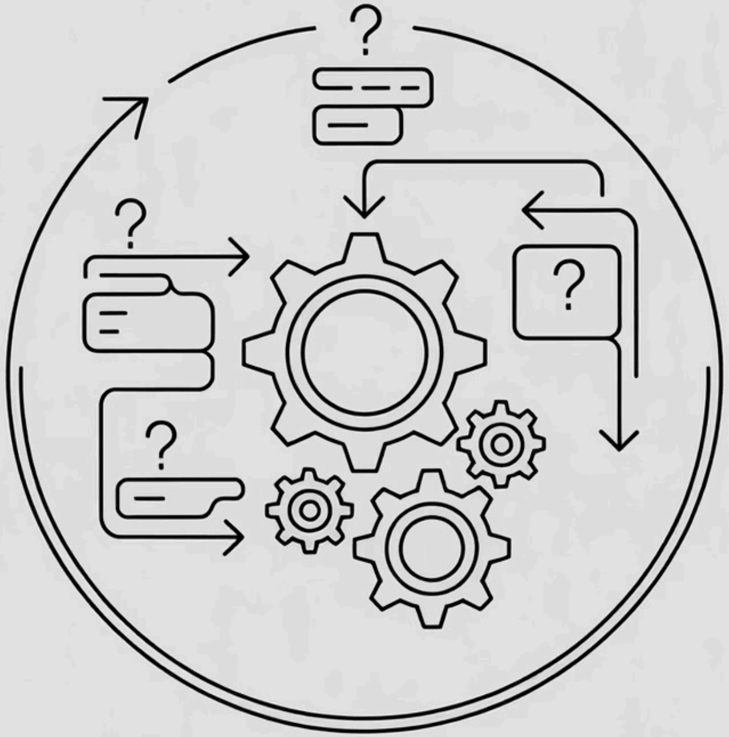
Within our defined time horizon, each component must undergo **preventive maintenance at least once**. This maintenance involves:

- **Duration (di)**: The time required to complete maintenance on component i
- **Execution time (ti)**: The specific point in time when maintenance begins

Resource Constraints

Maintenance activities are performed by a limited pool of **NRM repairmen**, with each repairman capable of maintaining **only one component at a time**. This creates a critical resource constraint that significantly impacts scheduling optimization.

Problem Formulation



Preventive Maintenance Cost Structure

The cost model for preventive maintenance incorporates multiple factors that influence the total expenditure and potential savings opportunities. Understanding these cost components is essential for developing optimization strategies.

Individual Component Maintenance Cost

For a single component *i*, the preventive maintenance cost is calculated as:

$$C_i^{PM} = C_{setup} + C_{i,material} + C_{i,labor}$$

Where setup costs include preparation, equipment, and administrative overhead; material costs cover replacement parts and consumables; and labor costs represent the time-based expense of maintenance personnel.

Separate Maintenance Approach

When each component is maintained independently, the total system maintenance cost is simply the sum of all individual component maintenance costs:

$$C_{separate} = \sum_{i=1}^{N_c} C_i^{PM}$$

This approach often results in higher overall costs due to redundant setup procedures and inefficient resource utilization.

Preventive Maintenance Cost Structure

The cost model for preventive maintenance incorporates multiple factors that influence the total expenditure and potential savings opportunities. Understanding these cost components is essential for developing optimization strategies.

Grouped Maintenance Approach

By grouping maintenance activities, significant cost savings can be realized:

$$C_{group} = C_{setup} + \sum_{i=1}^{N_c} C_{i,marginal}$$

This approach allows the setup cost to be shared across multiple components, with each component only incurring its marginal maintenance cost rather than the full individual cost.

Cost Savings Calculation

The financial benefit of grouping maintenance activities is quantified as:

$$\Delta C = C_{separate} - C_{group}$$

These savings primarily come from reduced setup costs, efficient resource utilization, decreased system downtime, and streamlined logistics operations.

System Reliability Modeling

Series System Reliability

In a series system, the overall reliability is the product of the individual component reliabilities, reflecting the fact that all components must function for the system to operate:

$$R_{system}(t) = \prod_{i=1}^{N_c} R_i(t)$$

This multiplicative relationship means that as the number of components increases, system reliability typically decreases, highlighting the importance of maintaining high individual component reliability through effective preventive maintenance.

For systems with a large number of components (high Nc), even small improvements in component reliability can significantly impact overall system performance.

Component Reliability Modeling

The reliability of each component is commonly modeled using the Weibull distribution, which offers flexibility in representing different failure patterns:

$$R_i(t) = \exp\left(-\left(\frac{t}{\eta_i}\right)^{\beta_i}\right)$$

Where:

- **ηi**: Scale parameter (characteristic life) for component i
- **βi**: Shape parameter for component i

The shape parameter β determines the failure rate behavior:

- **β < 1**: Decreasing failure rate (early failures/infant mortality)
- **β = 1**: Constant failure rate (exponential distribution)
- **β > 1**: Increasing failure rate (wear-out failures)

Accurate reliability modeling is essential for determining optimal maintenance intervals that balance the risk of failure against maintenance costs. Reliability data collection and analysis should be an ongoing process to refine and validate these models over time.

Maintenance Constraints

Resource Constraints

The number of available repairmen (NRM) creates a fundamental constraint on maintenance scheduling. Each repairman can only work on a single component at any given time, creating a resource allocation problem:

- Personnel availability fluctuations due to shifts, vacations, and training
- Specialized skill requirements for specific components
- Overtime considerations and associated cost premiums
- Productivity variations between maintenance teams

Time Horizon Constraints

All maintenance activities must be completed within the specified planning horizon (H), which may be determined by:

- Production schedules and planned shutdowns
- Regulatory compliance deadlines
- Component-specific reliability degradation rates
- Budget cycle limitations and fiscal year boundaries

Operational Constraints

Various operational factors impose additional constraints on maintenance scheduling:

- **Maintenance Windows**: Specific time periods when maintenance is permitted
- **Sequential Dependencies**: Required order of maintenance for interdependent components
- **System Access Limitations**: Safety protocols or production requirements that restrict component accessibility
- **Environmental Factors**: Weather conditions or temperature requirements for certain maintenance procedures

Logistic Constraints

The availability of necessary resources creates practical limitations:

- **Spare Parts Inventory**: Lead times and availability of critical components
- **Specialized Tools**: Limited quantities of diagnostic equipment or maintenance tools
- **Facility Space**: Physical workspace limitations for concurrent maintenance activities
- **External Contractor Availability**: Scheduling constraints for specialized service providers

Decision Variables

The optimization of preventive maintenance scheduling requires defining several key decision variables that collectively determine the complete maintenance plan. These variables form the mathematical foundation for the optimization model.

Assignment Variables

These variables determine which repairman is assigned to maintain each component:

- Binary assignment indicators (xi,r) for each component-repairman pair
- Considers repairman skill levels and component-specific expertise requirements
- Accounts for certification or qualification constraints
- Balances workload across the maintenance team

Timing Variables

These variables establish when each maintenance activity begins:

- Start time (si) for maintenance of component i
- Determines the sequence of maintenance activities
- Affects resource utilization efficiency
- Impacts system downtime patterns

Grouping Variables

These variables define which components are maintained together:

- Group membership indicators (gi)
- Enables setup cost sharing between components
- Affects overall maintenance duration
- Influences system reliability profile

Mathematical Representation

The complete decision vector can be represented as:

$$x = \{s_i, g_i, r_i \;\; \forall i \in \{1, \dots, N_c\}\}$$

Where:

- si: Start time for maintenance of component i
- gi: Group assignment for component i
- ri: Repairman assigned to component i

The dimensionality of this decision space grows rapidly with the number of components and repairmen, creating a combinatorial optimization challenge that typically requires advanced solution methods such as mixed-integer programming, genetic algorithms, or constraint programming.

Objective Function for Cost Minimization

This objective function balances multiple cost components to find the most economical maintenance strategy.

$$\min C_{total} = \sum_{i=1}^{N_c} C_i^{PM} - \Delta C_{group} + C_{downtime}$$

Preventive Maintenance Costs

The sum of all individual component maintenance costs:

$$\sum_{i=1}^{N_c} C_i^{PM}$$

These costs include labor, materials, equipment, and administrative expenses associated with each maintenance activity. Component-specific factors such as complexity, accessibility, and specialized skill requirements drive variations in these costs.

Grouping Cost Savings

The financial benefit achieved through maintenance grouping:

$$\Delta C_{group}$$

This term represents the cost reduction achieved by consolidating maintenance activities to share setup costs, reduce system downtime, and optimize resource utilization. The magnitude of these savings varies based on the efficiency of the grouping strategy and the inherent setup cost structure.

Downtime Cost Penalties

The economic impact of system unavailability during maintenance:

$$C_{downtime}$$

This cost component captures the production losses, opportunity costs, and other negative financial impacts associated with system shutdowns during maintenance. For critical systems, these costs can dominate the objective function and heavily influence the optimal scheduling strategy.

Constraints Formulation

The optimization model is subject to several fundamental constraints that define the feasible solution space. These mathematical expressions ensure that the resulting maintenance schedule is practically implementable.

Assignment Constraint

$$\sum_{r=1}^{N_{RM}} x_{i,r} = 1 \quad \forall i$$

This constraint ensures that each component is assigned to exactly one repairman. The binary variable xi,r equals 1 if component i is assigned to repairman r, and 0 otherwise. This prevents maintenance responsibility gaps or duplications.

Non-Overlap Constraint

$$s_i + d_i \leq s_j \ \vee \ s_j + d_j \leq s_i$$

This disjunctive constraint prevents a single repairman from being assigned to multiple simultaneous maintenance tasks. It ensures that for any two components assigned to the same repairman, either the first maintenance activity completes before the second begins, or vice versa.

Horizon Constraint

$$s_i + d_i \leq H \quad \forall i$$

This constraint ensures that all maintenance activities are completed within the planning horizon H. It prevents scheduling of maintenance beyond the available time window, which might represent a planned shutdown period or a regulatory compliance deadline.

Constraints Formulation: Additional Model Constraints

Resource Capacity

At any point in time t, the number of active maintenance activities cannot exceed the number of available repairmen:

$$\sum_{i=1}^{N_c} A_i(t) \leq N_{RM} \quad \forall t \in [0, H]$$

Where Ai(t) is an indicator function that equals 1 if component i is under maintenance at time t, and 0 otherwise.

Grouping Consistency

Components in the same maintenance group must be scheduled at the same time:

$$g_i = g_j \Rightarrow s_i = s_j \quad \forall i, j$$

This ensures that maintenance activities intended to share setup costs are actually performed simultaneously, enabling the modeled cost savings to be realized.